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The unscented Kalman Filter for the Estimation the States of The Boiler-Turbin Model

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Abstract— In many cases interesting dynamic are not linear by nature, so the traditional Kalman filter cannot be applied in estimating the state of such a system. In these kinds of systems, both the dynamics and the measurement processes can be nonlinear, or only one of them. In this paper, an extension to the traditional Kalman filter will be described, which can be applied for estimating nonlinear dynamic systems, that is called Unscented Kalman filter (UKF) based on the unscented transformation of the joint distribution. Then this method is used for the estimation of the states of the Boiler-Turbin Model. Simulation results show the effectiveness of this method.

Index Terms— Kalman Filter (KF), Unscented Kalman Filter (UKF), Extended Kalman Filter (EKF), Boiler- Turbin Model

1 Introduction

Estimating the state of a dynamic system is a fundamental problem in the process industries. State estimation often-play an important role in accomplishing this goal in process control and performance monitoring applications. Depending on the type of process and the operating region of the process, some processes can be approximated with alinear model and the KF (Kalman filter) can be used for state estimation. Theorically the kalman filter is an estimator for what is called the linear quadratic problem, which is the problem of estimating the state of a linear dynamic system, so for nonlinear dynamic, the most successful techniques for state estimation are Bayesian filters such as particle filters or extended and unscented Kalman filters [1]. Bayes filters recursively estimate posterior probability distributions over the state of a system. The key components of a Bayes filter are the prediction and observation models, which probabilistically describe the temporal evolution of the processand the measurements returned by the sensors, respect tively. Typically, these models are parametric descriptions of the involved processes. The most common way of applying the KF to a nonlinear system is in the form ofthe extended K alman filter (EKF). In the EKF, the pdf is propagated through a linear approximation of the system around the operating point at each time instant. In doing so, the EKF needs the Jacobian matrices which may be difficult to obtain for higher order systems, especially in the case of time-critical applications. Further, the linear approximation of the system at a given time instant may introduce errors in the state which may lead the state to diverge over time. In other words, the linear approximation may not be appropriate for some systems. In order to overcome the drawback EKF, other nonlinear state estimators have been developed such as the Unscented Kalman Filter (UKF) [2], the ensemble Kalman filter (EnKF) [8] and high order EKFs. The EnKF is especially designed for large scale systems, for instance, oceanographic models and reservoir models [3]. The UKF seems to be a promising alternative for process control applications [4-6].

The UKF propagates the pdf in a simple and effective way and it is accurate up to second order in estimating mean and covariance [8]. The present paper focuses on using the UKF for nonlinear state estimation in process systems and the performance is evaluated in comparison with the EKF.The paperproposes a simple method to incorporate state constraints in the UKF.

In a boiler-turbine unit, steam from the boiler enters the high-pressure cylinder of a condensing turbine and, after passing through it, returns to the boiler, entering through an intermediate superheater. The secondary superheated steam is fed into the medium-pressure cylinder of the turbine and then into the low-pressure cylinder and the condenser. The water is removed from the condenser by a pump. It then passes through the low-pressure and high-pressure feed-water heaters and a deaerator and enters the boiler. Usually, a boiler cannot operate at loads below a certain value for a number of reasons (for example, the conditions of cooling of the tubes of heating surfaces); therefore, some-times more steam is generated than is required for the turbine (for example, during the start-up of a unit) [7]. In such cases the excess steam is dumped into the condenser through a reduction device. Then Bell and Astrom produce a 3rd order non-linear MIMO model with fuel flow, control valve position, and feedwater flow as control inputs, and drum pressure, power output, and drum water level deviation as outputs. Boiler- turbin unit is a multivariable, time varying and nonlinear system with strong coupling between the parameters.

In this paper, the states of boiler- turbin are estimated with unscented kalman filter. In section 2, The Unscented kalman filter is introduced. Then in section 3, the Boiler- turbin unit is described. Section 4 discusses the simulation results followed by

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conclusions in section 5.

2 Unscented Kalman Filter

2.1 Unscented transform

The unscented transform (UT) (Julier et al., 1995; Julier and Uhlmann, 2004b;Wan and van der Merwe, 2001) can be used for forming a Gaussian approximation to the joint distribution of random variables x and y. In UT we deterministically choose a fixed number of sigma points, which capture the desired moments (at least mean and covariance) of the original distribution of x exactly.

After that we propagate the sigma points through the non-linear function g and estimate the moments of the transformed variable from them [8].

The advantage of UT over the Taylor series based approximation is that UT is better at capturing the higher order moments caused by the non-linear transform, as discussed in (Julier and Uhlmann, 2004b). Also the Jacobian and Hessian matrices are not needed, so the estimation procedure is in general easier and less error-prone. The unscented transform can be used to provide a Gaussian approximation for the joint distribution of variables x and y of the form

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim \boldsymbol{N}\left(\begin{pmatrix} m \\ \boldsymbol{\mu_u} \end{pmatrix}, \begin{pmatrix} P & \boldsymbol{C_u} \\ \boldsymbol{C_u^T} & \boldsymbol{s_u} \end{pmatrix}\right) \quad (1)$$

1. Compute the set of 2n + 1 sigma points from the columns of the matrix $\sqrt{(n+\lambda)P}$

$$x^{(0)} = m$$
$$x^{(i)} = m + [\sqrt{(n\lambda + P)}]_i, \; i = 1, 2, \ldots, n \quad (2)$$
$$x^{(i)} = m - [\sqrt{(n\lambda + P)}]_i, \; i = n+1, \ldots, 2n$$

and the associated weights:

$$W_m^{(0)} = \frac{\lambda}{n+\lambda}$$
$$(3)$$
$$W_c^{(0)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta)$$

Parameter λ is a scaling parameter, which is defined as

$$\lambda = \alpha^2 (n+k) - n \quad (4)$$

The positive constants α,β and k are used as parameters of the method.

2. Propagate each of the sigma points through non-linearity as

$$y^{(i)} = g\left(x^{(i)}\right), \; i = 0, 1, \ldots, 2n \quad (5)$$

3. Calculate the mean and covariance estimates for y as

$$(6) \quad \mu_u = \sum_{i=0}^{2n} w_m^{(i)} y^{(i)}$$
$$S_u = \sum_{0}^{i=2n} W_c^{(i)} (y^{(i)} - \mu_u)(y^{(i)} - \mu_u)^T$$

4. Estimate the cross-covariance between x and y as

$$C_u = \sum_{i=0}^{2n} W_c^{(i)} (x^{(i)} - m)(y^{(i)} - \mu_u)^T \quad (7)$$

The square root of positive definite matrix P is defined as $A = \sqrt{P}$ where

$$P = AA^T$$

To calculate the matrix A we can use, for example, lower triangular matrix of the Cholesky factorialization.

2.2 Unscented Kalman Filter

The unscented Kalman filter (UKF), makes use of the unscented transform described above to give a Gaussian approximation to the filtering solutions of non-linear optimal filtering problems of form

$$x_k = f\left(x_{k-1}, k-1\right) + q_{k-1} \quad (8)$$

$$y_k = h\left(x_k, k\right) + r_k$$

Where $x_k \in R^n$ is the state, $y_k \in R^m$ is the measurement, $q_{k-1} \approx N(0, Q_{k-1})$ is the Gaussain Noise process, $r_k \approx N(0, R_k)$ is the Gaussian measurement noise. The prediction and update steps are in the following way:

Prediction: Compute the predicted state mean m_k^- and the predicted covariance P_k^- as

$$X_{k-1} = [m_{k-1} \ldots\ldots m_{k-1}] + \sqrt{c}\left[0 \sqrt{P_{k-1}} \sqrt{P_{k-1}}\right]$$

$$\hat{X}_k = f(X_{k-1}, k-1)$$

$$m_k^- = \hat{x}_k w_m \quad (9)$$

$$P_k^- = \hat{x}_k w[\hat{x}_k]^T + Q_{k-1}$$

Update: Compute the predicted mean μk and covariance of the measurement Sk, and the cross-covariance of the state and measurement Ck:

$$X_k^- = [m_k^-, \ldots\ldots, m_k^-] + \sqrt{c}\left[0 \sqrt{P_k^-} \sqrt{P_k^-}\right]$$

$$Y_k^- = h(x_k^-, k)$$

$$\mu_k = Y_k^- w_m$$

$$S_k = Y_k^- w[Y_k^-]^T + R_k \quad (10)$$

$$C_k = X_k^- W[Y_k^-]^T$$

Then compute the filter gain Kk and the updated state mean mk and covariance Pk:

$$K_k = C_k S_k^{-1} \quad (11)$$

$$P_k = P_k^- - K_k S_k K_k^T$$

$$m_k = m_k^- + K_k[y_k - \mu_k]$$

2.3 Augmented Unscented Kalman filter

It is possible to modify the UKF procedure described above by forming an augmented state variable, which concatenates the state and noise components together, so that the effect of process and measurement noises can be used to better capture the odd-order moment information. This requires that the sigma points generated during the predict step are also used in the update step, so that the effect of noise terms are truly propagated through the nonlinearity (Wu et al., 2005). If, however, we generate new sigma points in the update step the augmented approach give the same results as the non-augmented, if we had assumed that the noises were additive. If the noises are not in the additive form, the augmented version should produce more accurate estimates than the non-augmented version, even if new sigma points are created during the update step.

So, the prediction and updating steps are in the form of:

Prediction:

$$\tilde{x}_{k-1} = [\tilde{m}_{k-1} \ldots\ldots \tilde{m}_{k-1}] + \sqrt{c}\left[0 \sqrt{\tilde{P}_{k-1}} - \sqrt{\tilde{P}_{k-1}}\right]$$

$$\tilde{m}_{k-1} = \left[m_{k-1}^T \, 0 \, 0\right]^T \; \tilde{P}_{k-1} = \begin{pmatrix} P_{k-1} & 0 & 0 \\ 0 & Q_{k-1} & 0 \\ 0 & 0 & R_{k-1} \end{pmatrix} \quad (12)$$

So:

$$\hat{x}_k = f\left(X_{k-1}^x, X_{k-1}^q, k-1\right)$$

$$m_k^- = \hat{x}_k w_m \quad (13)$$

$$P_k^- = \hat{x}_k w[\hat{x}_k]^T$$

In this way, the noise must be in additive form.

Update:

$$Y_k^- = h(x_k^-, x_{k-1}^r, k)$$

$$\mu_k = Y_k^- w_m \quad (14)$$

$$S_k = Y_k^- w[Y_k^-]^T$$

$$C_k = \hat{x}_k w[Y_k^-]^T$$

Where we have denoted the component of sigma points corresponding to measurement noise with matric x_{k-1}^r. Like the state transition function f also the measurement function h is now augmented to incorporate the effect of measurement noise, which is passed as a second parameter to the function. Then compute the filter gain K_k and the updated state mean m_k and covariance P_k:

$$K_k = C_k S_k^{-1}$$

$$m_k = m_k^- + K_k[y_k - \mu_k] \quad (15)$$

$$P_k = P_k^- - K_k S_k K_k^T$$

Note that non-augmented form UKF is computationally less demanding than augmented form UKF, because it creates a

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smaller number of sigma points during the filtering procedure. Thus, the usage of the non-augmented version should be preferred over the non-augmented version, if the propagation of noise terms doesn't improve the accuracy of the estimates.

3. The nonlinear model of Boiler-Turbin unit

The model is based on the boiler-turbine plant P16/G1 at the Sydvensb Kraft AB plant in Mamlo, Sweden. The boiler is oil-fired and the rated power is 160 *MW*. Data acquired during a series of experiments in 1969 form the basis for the system identification. Both physics and empirical methods were used to produce this boiler-turbine dynamic model [9].

Since 1969, the model has undergone a number of alterations. Subsequent improvements in the plant model have resulted in better models that yield improved predictive abilities for the plant.

This resulted in a 2nd order non-linear system of differential equations with fuel flow and control valve setting as the control variables and drum pressure and power Output as the Output variables.

Then Bell and Astrom produce a 3rd order non-linear MIMO model with fuel flow, control valve position, and feedwater flow as control inputs, and drum pressure, power output, and drum water level deviation as outputs.

Boiler- turbin unit is a multivariable, time varying and nonlinear system with strong coupling between the parameters. The dynamic of the system is in the form of:

$$\begin{cases} x_1' = -0.0018u_2 x_1^{9/8} + 0.9u_1 - 0.15u_3 \\ x_2' = (0.073u_2 - 0.016)x_1^{9/8} - 0.1x_2 \\ x_3' = (141u_3 - (1.1u_2 - 0.19)x_1)/85 \\ y_1 = x_1 \\ y_2 = x_2 \\ y_3 = 0.05(0.13073x_3 + 100a_{cs} - 67.975) \end{cases} \quad (16)$$

Where

X_1, x_2, x_3 are drum pressure (kg/cm^2), power output(MW), fluid density (kg/m3) respectively. The normalized inputs to the system u_1 = fuel flow valve position, u_2 = steam control valve position, and u_3 = feedwater flow valve position, and all valve position variables are constrained to lie in the interval [0, 1]. The outputs to the system are y_1 (drum pressure), y_2 (output level), y_3 is the drum water level in meter. And the variable a_{cs}, q_e are steam quality and the evaporation rate (kg/s):

$$a_{cs} = \frac{(1 - 0.001538x_3)(0.8x_1 - 25.6)}{x_3(1.0394 - 0.0012304x_1)} \quad (17)$$

$$q_e = (0.854u_2 - 0.147)x_1 + 45.59u_1 - 2.514u_3 - 2.096$$

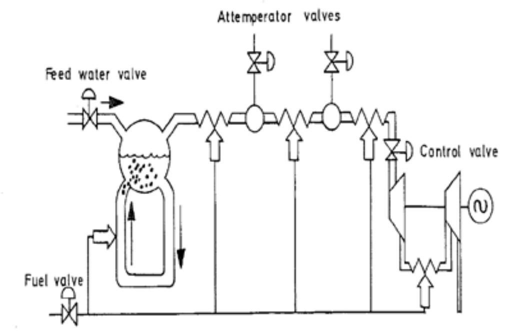

Fig.1 Schematic diagram of the Boiler-Turbin unit

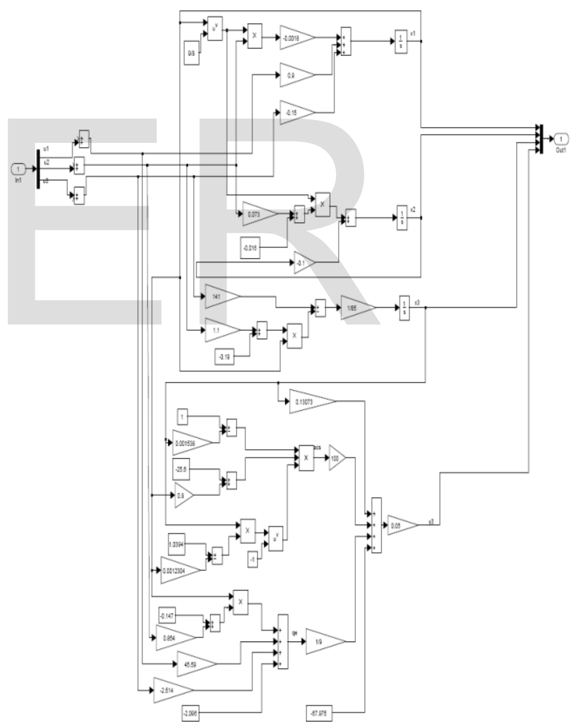

Fig 2. The simulation of the Boiler- Turbin unit

Figure 1, shows the schematic diagram of a boiler turbin unit, and also in figure 2, the simulation of boiler turbin is brought that is simulated in Matlab simulation.

4. Simulation Results

For the Boiler- Turbin, as discussed in the previous section, the states x_1, x_2, x_3 are estimated with Augmentd Kalman Filter. In this way, noise must be added to the system. In this method, linearization is not needed. So this method is better than the traditional kalman filter.

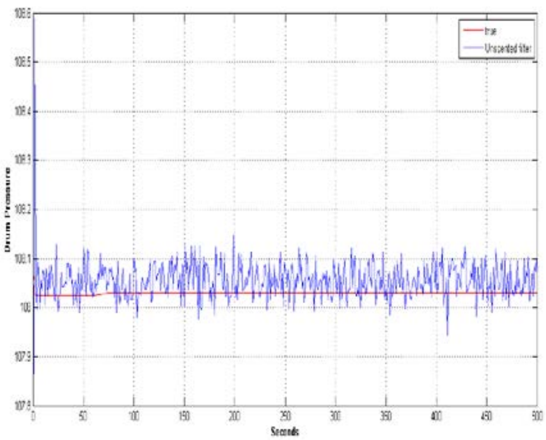

Fig.3 The estimation of the state x_1

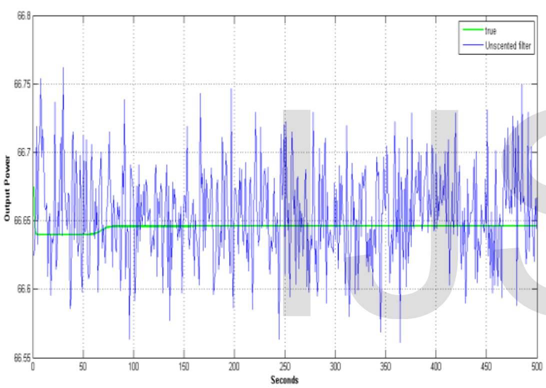

Fig.4 The estimation of the state x_2

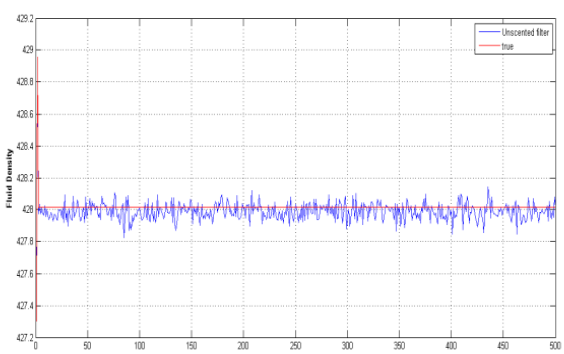

Fig.5 The estimation of the state x_3

In the most papers the equiblirium for the Boiler_ Turbin system $[x_1,x_2,x_3,u_1,u_2,u_3]=[108,66.65,428,0.34,0.69,0.433]$ is considered. The above figures show that the estimation of the states x_1, x_2, x_3 are accurate.

5. Conclusion

In this paper, Unscented Kalman filter is discussed, and this method is considered for boiler- turbin system. In compare with traditional kalman filter, linearization is not needed. When linearization is used, only the equiblirium point is examined, but Unscented kalman filter estimates the states around all the points without any linearization and is more accurate than the kalman filter and the time for simulation in less than the other method.

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